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Thermal transpiration in a circular capillary with a small temperature difference

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The results of an experimental investigation of the thermal transpiration effect (the thermomolecular pressure difference or t.p.d. effect) in a single glass capillary with a length-to-radius ratio of 250 are presented. The temperatures of the gas in the 'cold' and 'hot' chambers were 273·2 °K and 293 °K, respectively. A modified relative method has been used. To measure the t.p.d. effect, a capacitance differential digital micromanometer with sensitivity $4·5 \times 10^{-5} \mathrm{N}/(\mathrm{m^2\,Hz})$ was used. The gases investigated were He, Ne, Ar, Xe, H_2, D_2, N_2, CO_2, CH_4 and SF_6. It was discovered that in the intermediate flow regime the thermal-creep flow rate does not depend on the (non-isothermal) tangential momentum accommodation coefficient. From the experimental data on the viscous slip flow regime, the Eucken factors and the accommodation coefficients are calculated. For inert gases the Eucken factor is found to be equal to 2·5 within the experimental error, while the accommodation coefficients differ significantly from unity.

1. Introduction

The pressure difference in a closed system filled with gas at a fixed temperature difference is called the thermomolecular pressure difference effect (the t.p.d. effect). From the practical point of view, the correct estimation of this effect is important when the gas pressure in the working chamber is measured with a device which is in different temperature conditions. The investigation of this effect is also of interest because it gives an important opportunity to study both qualitatively and quantitatively the interaction of gas molecules with solid surfaces.

The t.p.d. effect, first discovered by Reynolds (1879) and studied by Maxwell (1953) and Knudsen (1910), continues to attract the attention of theorists and experimentalists. At present there exists a large amount of experimental material obtained by different methods for a wide range of Knudsen numbers at various temperature differences (Itterbeek & Grande 1947; Bennet & Tompkins 1957; Podgurski & Davis 1961; Edmonds & Hobson 1965; Hobson 1969; McConville, Taylor & Watkins 1970; Ganzi & Sandler 1971; Annis 1972; Borisov, Kulev, Porodnov & Suetin 1973; Borisov, Kulev, Porodnov, Suetin & Barashkin 1973).

Despite the great number of experimental results there exists no correct theory for the t.p.d. effect at arbitrary Knudsen numbers. Results have been obtained only for free-molecular flow (Knudsen 1910) and the continuum regime (Deryagin & Bakanov 1962; Suetin 1966). To describe experimental data over the whole range of

Knudsen numbers the semiempirical Liang (1955) formula as modified in Bennet & Tompkins (1957) has been widely used.

In recent years an attempt has been made to describe the t.p.d. effect by solving the Boltzmann equation or a model for both plane (Borisov *et al.* 1970; Gorelov & Cogan 1970; Loyalka 1971; Chernjak *et al.* 1975*a*, *b*; Loyalka & Cipolla 1971) and cylindrical geometries (Sone & Yamamoto 1968; Loyalka 1969; Chernjak, Porodnov & Suetin 1973, 1974). However, as a rule, completely diffuse scattering of the gas molecules by the surface was assumed in the theoretical considerations. The use of diffuse–specular boundary conditions makes it possible not only to describe qualitatively the observed results (Chernjak *et al.* 1975*a*, *b*; Loyalka & Cipolla 1971) but also to compare quantitatively theoretical and experimental results for cylindrical capillaries for an arbitrary flow regime (Chernjak *et al.* 1973). In a recent paper (Loyalka 1975) an analogous problem has been solved by direct numerical solution of the integral transfer equations.

It should be noted that, since the t.p.d. effect is small and the experimental technique is not sufficiently efficient, the t.p.d. effect has so far been measured at temperature differences ΔT comparable to the mean gas temperature T_0, i.e. $\Delta T/T_0 \sim 1$. This makes comparison of experimental data with theoretical predictions, which are obtained, as a rule, for the case of a small perturbation of the velocity distribution function (i.e. $\Delta T/T_0 \ll 1$), exceedingly difficult.

In the present paper some theoretical results and experimental data on the t.p.d. effect on inert and polyatomic gases in a single cylindrical capillary with a small temperature difference are given. From a comparison of the theoretical and experimental results, such characteristics as the flow rates, the thermal-creep constants, the Eucken factors and the accommodation coefficients are determined.

2. Theory

2.1. *Basic formulae for the t.p.d. effect*

In Chernjak *et al.* (1973, 1974, 1975*a*, *b*) the formulation of the t.p.d. problem and the solution of the kinetic equation (BGK model) by an integral method with Maxwellian diffuse–specular boundary conditions are described in detail. Therefore, here we describe only those details which are necessary to confirm the correctness of the theoretical formula used to describe the experimental data and to determine the corresponding parameters.

Let the temperature gradient be along the (z) axis of a capillary with radius R. This capillary connects two chambers. Owing to thermal creep flow, the gas pressure increases in the 'hot' chamber whereas it decreases in the 'cold' chamber. As a result of the pressure gradient, a Poiseuille flow directed towards the 'cold' chamber appears in the capillary. The thermal creep flow and the Poiseuille flow can be characterized by average (over the capillary cross-section) flow rates $\overline{U}_T$ and $\overline{U}_P$ which are related to the non-dimensional flow rates Q_T and Q_P through the cross-section at z by

$$Q_T = \frac{2\langle u_T \rangle}{\tau S}, \quad Q_P = -\frac{2\langle u_P \rangle}{\nu S}, \quad \langle u_{T,P} \rangle = \left(\frac{m}{2kT}\right)^{\frac{1}{2}} \overline{U}_{T,P}, \quad \tau = \frac{R}{T}\frac{dT}{dz}, \quad \nu = \frac{R}{P}\frac{dP}{dz},$$

$$\tag{1}$$

where S is the cross-sectional area of the capillary and τ and ν are the logarithmic temperature and pressure gradients, respectively.

It is evident that in the steady state the total average gas flow rate is zero, i.e. $\langle u_T \rangle + \langle u_P \rangle = 0$. In this case, it is easy to obtain from (1) the differential formula for the t.p.d. effect:

$$\frac{dP}{P} = \gamma \frac{dT}{T}, \quad \gamma = \frac{Q_T}{Q_P}. \tag{2}$$

In the experiments the gas pressures P_1 and P_2 and temperatures T_1 and T_2 in the 'cold' and 'hot' chambers, respectively, were always measured. Therefore, for the t.p.d. effect the integrated formula is of practical interest. This formula can be obtained by integrating (2) along the capillary between the corresponding limits for the temperature and pressure in the form

$$\frac{P_2}{P_1} = \exp\left[\int_{T_1}^{T_2} \gamma \frac{dT}{T}\right]. \tag{3}$$

It should be noted that (3) is valid for the case $\Delta P/P_0 \equiv 2(P_2 - P_1)/(P_2 + P_1) \ll 1$. Since the γ, according to (2), is determined by the ratio of the non-dimensional flow rates Q_T and Q_P, which depend on the temperature and pressure, in general γ cannot be taken outside the integral sign in (3). Only when $T_1 \approx T_2$ does (3) give

$$P_2/P_1 = (T_2/T_1)^\gamma. \tag{4}$$

This formula is the generalization to the arbitrary flow regime of the well-known Knudsen formula, which was obtained in the free-molecular flow regime with $\gamma = \frac{1}{2}$ (Knudsen 1910).

Thus the main difficulty in the description of this effect is the determination of Q_T and Q_P and the integration of (3).

2.2. *Integral transport equations*

The Q_T and Q_P flows can be found from the solution of the basic kinetic equation for the perturbation function ϕ by the BGK model for the intermolecular collisions operator. The Poiseuille flow and the thermal-creep flow are investigated individually (as in Borisov *et al.* 1970; Chernjak *et al.* 1973, 1974, 1975a, b; Sone & Yamamoto 1968; Loyalka 1971; Loyalka & Cipolla 1971) in the steady state for slow flows, when flows are proportional to the macroscopic value gradients. It is assumed that, since both flow rates are small, there will be negligible interaction between the solutions when they are combined to give zero net flow. Each flow can be described by means of corresponding kinetic equations such as

$$\mathbf{C} \cdot \partial\phi_P/\partial\mathbf{r} + C_z\nu = \delta_P[2C_z u_P(\mathbf{r}) - \phi_P(\mathbf{r}, C_z, \mathbf{C})], \tag{5a}$$

$$\mathbf{C} \cdot \partial\phi_T/\partial\mathbf{r} + C_z\tau(C^2 + C_z^2 - \tfrac{5}{2}) = \delta_T[2C_z u_T(\mathbf{r}) - \phi_T(\mathbf{r}, C_z, \mathbf{C})], \tag{5b}$$

$$\delta_{P,T} = \tfrac{1}{2}\pi^{\frac{1}{2}}(R/\lambda_{P,T}). \tag{5c}$$

Here $(\mathbf{C}, C_z) = (m/2kT)^{\frac{1}{2}}(\mathbf{v}, v_z)$ are the non-dimensional molecular velocities, ϕ_P and ϕ_T are the perturbation functions and δ_P and δ_T are the rarefaction parameters characterizing the Poiseuille and thermal-creep flows, respectively.

The mean free paths λ_P and λ_T in (5c) are given by

$$\lambda_P = \pi^{\frac{1}{2}} \left(\frac{\eta}{P}\right)\left(\frac{kT}{2m}\right)^{\frac{1}{2}}, \quad \lambda_T = \frac{4}{5}\frac{\pi^{\frac{1}{2}}}{2}\left(\frac{\kappa}{P}\right)\left(\frac{mT}{2k}\right)^{\frac{1}{2}}, \tag{6}$$

where η is the viscosity coefficient and κ is the translational part of the heat conductivity coefficient. Such a choice for the rarefaction parameters is necessary to give the correct value of the Prandtl number, which for rarefied gases is $Pr = \frac{2}{3}$ (Hirschfelder, Curtiss & Bird 1961, p. 30; Cogan 1967, p. 131; Loyalka & Cipolla 1971). Note that use of a single parameter δ to describe the t.p.d. effect by the BGK model leads to an incorrect value of the Prandtl number (namely $Pr = 1$) and to an incorrect coefficient in the formula for the thermal-creep flow rate in the viscous flow regime.

From (5c) and (6) it is easy to obtain the relation between the parameters δ_P and δ_T:

$$\delta_P = \tfrac{3}{5}f_{\text{tr}}\,\delta_T, \tag{7}$$

where f_{tr} is the translational Eucken factor (Hirschfelder *et al.* 1961, p. 398; Loyalka & Cipolla 1971).

Chernjak *et al.* (1973) have solved the system (5) under the assumption of arbitrary tangential momentum accommodation of molecules incident on the wall. They obtained results for values of the parameter δ ranging from 0 to 10 and values of the accommodation coefficient ϵ ranging from 1 to 0·90. Note that Chernjak *et al.* (1973) used the single parameter δ and the single accommodation coefficient ϵ to describe both the Poiseuille flow and the thermal-creep flow.

As mentioned above, here we use two rarefaction parameters δ_P and δ_T and two tangential momentum accommodation coefficients ϵ and α for the Poiseuille flow and the thermal-creep flow, respectively. The two accommodation coefficients ϵ and α have the same physical meaning. The difference between these coefficients lies in the fact that ϵ describes the gas–surface interaction when the temperatures of the gas and the surface are the same (the Poiseuille flow), whereas α describes the gas–surface interaction when the temperatures of the gas and the surface are different (the thermal-creep flow).

As in Suetin *et al.* (1973), one can use the definition of the macroscopic gas flow rates u_P and u_T and introduce the new functions

$$\psi_P = 1 - \frac{2\delta_P}{\nu}u_P, \quad \psi_T = 1 + \frac{2\delta_T}{\tau}u_T. \tag{8}$$

Then, from (5a, b), it is not difficult to obtain the following integral equations through integration along the direction of the characteristic velocity **c** and use of the Maxwellian boundary conditions as in Suetin *et al.* (1973):

$$\psi_P(r) = 1 + \frac{\delta_P}{\pi}\int_0^{2\pi} d\alpha_1 \int_0^{S_0} dS \int_0^{\infty} \psi_P(r')\exp(-c^2 - \delta_P c^{-1}S)\,dc$$

$$+ \frac{\delta_P}{\pi}\int_0^{2\pi} d\alpha_1 \int_0^{S_0} dS \int_0^{\infty} \psi_P(r')\frac{(1-\epsilon)\exp[-c^2 - \delta_P c^{-1}(S+S_0)]}{1-(1-\epsilon)\exp(-\delta_P c^{-1}l)}\,dc, \tag{9a}$$

$$\psi_T(r) = 1 - \frac{\delta_T}{\pi}\int_0^{2\pi} d\alpha_1 \int_0^{S_0} dS \int_0^{\infty} [c^2 - \psi_T(r')]\exp(-c^2 - \delta_T c^{-1}S)\,dc$$

$$- \frac{\delta_T}{\pi}\int_0^{2\pi} d\alpha_1 \int_0^{l} dS \int_0^{\infty} [c^2 - \psi_T(r')]\frac{(1-\alpha)\exp[-c^2 - \delta_T c^{-1}(S+S_0)]}{1-(1-\alpha)\exp(-\delta_T c^{-1}l)}\,dc. \tag{9b}$$

ϵ / δ_P	1	0·98	0·96	0·94	0·92	0·90	0·88	0·86	0·84	0·82	0·80
0·01	1·476	1·534	1·594	1·657	1·723	1·791	1·862	1·936	2·026	2·099	2·187
0·02	1·460	1·516	1·574	1·635	1·693	1·764	1·833	1·904	1·983	2·059	2·144
0·04	1·438	1·492	1·547	1·605	1·666	1·728	1·794	1·861	1·933	2·002	2·085
0·06	1·423	1·475	1·529	1·585	1·643	1·703	1·766	1·832	1·899	1·979	2·045
0·08	1·412	1·462	1·515	1·569	1·626	1·685	1·746	1·810	1·873	1·945	2·014
0·1	1·403	1·452	1·503	1·556	1·611	1·668	1·728	1·790	1·853	1·921	1·992
0·2	1·381	1·426	1·474	1·523	1·574	1·627	1·682	1·480	1·806	1·885	1·931
0·4	1·378	1·420	1·465	1·510	1·562	1·615	1·668	1·719	1·768	1·827	1·888
0·6	1·397	1·437	1·479	1·523	1·568	1·621	1·671	1·721	1·772	1·828	1·888
0·8	1·424	1·464	1·505	1·547	1·592	1·638	1·687	1·737	1·791	1·846	1·904
1	1·457	1·496	1·536	1·578	1·622	1·668	1·715	1·765	1·818	1·873	1·930
2	1·655	1·693	1·732	1·773	1·816	1·861	1·907	1·956	2·007	2·060	2·116
3	1·877	1·914	1·953	1·994	2·037	2·081	2·127	2·176	2·227	2·280	2·336
4	2·107	2·145	2·184	2·225	2·267	2·312	2·358	2·406	2·458	2·511	2·567
5	2·343	2·381	2·420	2·461	2·504	2·548	2·594	2·643	2·694	2·747	2·803
6	2·583	2·620	2·659	2·700	2·742	2·787	2·833	2·881	2·934	2·987	3·003
7	2·824	2·862	2·901	2·942	2·984	3·029	3·075	3·123	3·167	3·229	3·285
8	3·067	3·105	3·144	3·185	3·228	3·272	3·319	3·375	3·420	3·473	3·529
9	3·312	3·349	3·389	3·430	3·472	3·517	3·563	3·612	3·664	3·718	3·778
10	3·557	3·595	3·642	3·675	3·717	3·761	3·808	3·856	3·910	3·963	4·019

TABLE 1. The non-dimensional Poiseuille flow rate Q_P as the function of the rarefaction parameter δ_P and the accommodation coefficient ϵ.

Here $S_0 = r\cos\alpha_1 + (1 - r^2\sin^2\alpha_1)^{\frac{1}{2}}$ and $l = 2(1 - r^2\sin^2\alpha_1)^{\frac{1}{2}}$ (see figure 1 in Suetin *et al.* 1973).

The procedure for solving the integral equation (9a), which was described in detail in Suetin *et al.* (1973), was also used to solve (9b). All we need to add is that we used the Bubnov–Galerkin method in the second and the third approximations as well as basic functions of the form

$$\psi_i^{(2)} = A_i + B_i r^2, \quad \psi_i^{(3)} = A_i + B_i r^2 + C_i r^4 \quad (i = P, T). \tag{10}$$

2.3. *Numerical results*

As the expressions obtained are very cumbersome we omit all intermediate calculations, including the coefficients in the second and third approximations, which are analogous to the coefficients a_{ij} in Suetin *et al.* (1973).

All calculations were made on the computer with an accuracy of 0·1 %. The results for Q_P and Q_T in the second approximation for different values of δ_P and δ_T and the accommodation coefficients ϵ and α are given in tables 1 and 2. The values of Q_P and Q_T in the second and third approximations practically coincide in the free-molecular and nearly-free-molecular flow regimes (the difference lies within the limits of the accuracy of the calculations). The difference in the results for the thermal-creep flow is less than 1 % in the viscous flow regime.

The calculations show that in the intermediate flow regime ($\delta_T = 2·51$) the non-dimensional thermal-creep flow Q_T is independent of the value of the accommodation coefficient α.

It should be noted that the results obtained by Loyalka (1975) coincide with ours

α / δ_T	1	0·96	0·92	0·88	0·84	0·80	0·76	0·72	0·68	0·64	0·60
0·01	0·7178	0·7730	0·8325	0·8968	0·9779	1·048	1·138	1·245	1·348	1·464	1·597
0·02	0·6939	0·7467	0·8013	0·8600	0·9278	0·9970	1·072	1·158	1·249	1·349	1·462
0·04	0·6629	0·7075	0·7553	0·8064	0·8629	0·9226	0·9871	1·056	1·133	1·216	1·315
0·06	0·6373	0·6774	0·7202	0·7659	0·8155	0·8687	0·9129	0·9851	1·052	1·126	1·200
0·08	0·6159	0·6525	0·6914	0·7328	0·7773	0·8236	0·8678	0·9288	0·9879	1·050	1·122
0·1	0·5975	0·6310	0·6665	0·7044	0·7428	0·7857	0·8319	0·8812	0·9350	0·9921	1·055
0·2	0·5293	0·5532	0·5784	0·6048	0·6349	0·6621	0·5927	0·7481	0·7581	0·7547	0·8296
0·4	0·4462	0·4607	0·4757	0·4913	0·5077	0·5248	0·5425	0·5613	0·5808	0·6018	0·6231
0·6	0·3923	0·4018	0·4117	0·4219	0·4325	0·4431	0·4549	0·4665	0·4789	0·4914	0·5050
0·8	0·3526	0·3592	0·3660	0·3729	0·3802	0·3876	0·3952	0·4030	0·4111	0·4194	0·4282
1	0·3215	0·3261	0·3308	0·3356	0·3408	0·3459	0·3512	0·3566	0·3622	0·3679	0·3738
2	0·2266	0·2273	0·2280	0·2288	0·2293	0·2300	0·2308	0·2315	0·2323	0·2330	0·2338
3	0·1759	0·1755	0·1751	0·1748	0·1744	0·1740	0·1736	0·1732	0·1728	0·1724	0·1720
4	0·1437	0·1430	0·1422	0·1415	0·1408	0·1401	0·1394	0·1386	0·1379	0·1372	0·1364
5	0·1213	0·1205	0·1197	0·1188	0·1180	0·1172	0·1164	0·1156	0·1147	0·1139	0·1130
6	0·1048	0·1040	0·1031	0·1023	0·1016	0·1007	0·0999	0·0991	0·0982	0·0974	0·0965
7	0·0923	0·0915	0·0906	0·0898	0·0891	0·0883	0·0874	0·0866	0·0858	0·0850	0·0842
8	0·0823	0·0816	0·0808	0·0800	0·0793	0·0785	0·0777	0·0770	0·0762	0·0754	0·0746
9	0·0743	0·0736	0·0728	0·0721	0·0714	0·0707	0·0700	0·0692	0·0685	0·0677	0·0670
10	0·0677	0·0670	0·0663	0·0655	0·0649	0·0643	0·0636	0·0629	0·0622	0·0615	0·0608

TABLE 2. The non-dimensional thermal-creep flow rate Q_T as the function of the rarefaction parameter δ_T and the accommodation coefficient α.

when $\alpha = \epsilon = 1$ but differ significantly when $\alpha < 1$ and $\epsilon < 1$. This difference is due to an error made by Loyalka (1975): he wrote the second and third terms on the right sides of (9 a, b) as a single term in which the integral over S had limits 0 and l. This is not correct.

We should like to note that equation (17) in Suetin et al. (1973) is given formally for simplicity. However, in all calculations Suetin et al. (1973) solved the integral equation in the form (9 a).

In the viscous slip flow regime ($\delta_{P,T} \gg 1$) asymptotic analysis of the results of the Bubnov–Galerkin method for the second approximation gave the following formulae for the flow rates Q_P and Q_T:

$$Q_P(\delta_P \gg 1) = \frac{\delta_P}{4} + \frac{2-\epsilon}{\epsilon}\frac{\pi^{\frac{1}{2}}}{2}\left(1 + \frac{4-\pi}{2\pi}\epsilon\right)$$
$$+ \left[\frac{(\pi-4^2)}{\pi} + \left(\frac{17\pi - 32 - 2\pi^2}{2\pi}\right)\epsilon + \left(\frac{(\pi-4)^2}{4\pi}\right)\epsilon^2\right]\frac{1}{\delta_P} + \frac{D_P}{\delta_P^2} + O\left(\frac{1}{\delta_P^3}\right), \quad (10a)$$

$$Q_T(\delta_T \gg 1) = \left(\frac{1}{2} + \frac{\alpha}{4}\right)\frac{1}{\delta_T} - \frac{\alpha}{4\pi^{\frac{1}{2}}}[2(\pi-1) + (4-\pi)\alpha]\frac{1}{\delta_T^{\frac{3}{2}}}$$
$$- \left[\frac{15\pi-32}{8\pi}\alpha - \left(\frac{83\pi - 160 - 8\pi^2}{16\pi}\right)\alpha^2 - \left(\frac{(\pi-4)^2}{4\pi}\right)\alpha^3\right]\frac{1}{\delta_T^3} + \frac{D_T}{\delta_T^4} + O\left(\frac{1}{\delta_T^5}\right). \quad (10b)$$

Here
$$\pi^{\frac{1}{2}}D_P = 10\pi - 72 + \frac{128}{\pi} + \frac{3\pi}{8}F(\epsilon) + \left(-\frac{192}{\pi} + \frac{245}{2} - 23\pi + \pi^2 - \frac{3\pi}{8}F(\epsilon)\right)\epsilon$$
$$+ \left(\frac{96}{\pi} - 68 + 15\pi - \pi^2 + \frac{3\pi}{32}F(\epsilon)\right)\epsilon^2 + \left(-\frac{16}{\pi} + 12 - 3\pi + \frac{\pi^2}{4}\right)\epsilon^3,$$

$$\pi^{\frac{1}{2}}D_T = \left(\frac{32}{\pi} - 25 + 4\pi - \tfrac{45}{16}F(\alpha) + \tfrac{105}{16}G(\alpha)\right)\alpha$$

$$+ \left(-\frac{96}{\pi} + 11\tfrac{5}{2} - \frac{17\pi}{2} + \tfrac{15}{16}F(\alpha) - \frac{9\pi}{32}G(\alpha)\right)\alpha^2$$

$$+ \left(\frac{72}{\pi} - 48 + \frac{19\pi}{2} - \frac{\pi^2}{2} - \tfrac{3}{4}F(\alpha) + \frac{9\pi}{32}G(\alpha)\right)\alpha^3$$

$$+ \left(-\frac{16}{\pi} + 12 - 3\pi + \frac{\pi^2}{4}\right)\alpha^4,$$

where

$$F(\mu) = 1 - \frac{1-\mu}{2^2} + \frac{(1-\mu)^2}{3^3} + \dots,$$

$$G(\mu) = 1 + \frac{1-\mu}{2^3} + \frac{(1-\mu)^2}{3^3} + \dots.$$

The first two terms of the asymptotic expansions of the solutions for $\delta_{P,T} \gg 1$ are

$$Q_P(\delta_P \gg 1) \approx \frac{\delta_P}{4} + \frac{2-\epsilon}{\epsilon}\frac{\pi^{\frac{1}{2}}}{2}\left(1 + \frac{4-\pi}{2\pi}\epsilon\right), \tag{11a}$$

$$Q_T(\delta_T \gg 1) \approx \left(\frac{1}{2} + \frac{\alpha}{4}\right)\frac{1}{\delta_T} - \frac{3\alpha}{4\pi^{\frac{1}{2}}}[2(\pi-3) + (4-\pi)\alpha]\frac{1}{\delta_T^2}. \tag{11b}$$

The higher-order terms in (11*a*, *b*) are difficult to obtain because of a considerable increase in the calculation required. Comparison of (10*a*, *b*) and (11*a*, *b*) shows that the convergence of the solution method used is better for the Poiseuille flow than for the thermal-creep flow. Comparison of the results calculated from the asymptotic formulae (10*a*, *b*) with the numerical results (tables 1 and 2) shows that these results coincide within 1 % when $\delta_P \geqslant 2$ and $\delta_T \geqslant 4$ and within 0·1 % when $\delta_P \geqslant 4$ and $\delta_T \geqslant 8$.

3. Experimental technique

In the present study the t.p.d. effect was measured by a modified relative method with a differential capacitance digital micromanometer (Borisov, Kulev, Porodnov & Suetin 1973; Borisov, Kulev, Porodnov, Suetin & Barashkin 1973) of sensitivity $\sim 4 \times 10^{-5} \, \text{N}/(\text{m}^2 \, \text{Hz})$.

The experimental set-up, shown in figure 1, differs from that in Borisov, Kulev, Porodnov & Suetin (1973) only in its construction and in the position of the bypass stopcock. This arrangement allows a large ratio (~ 2000) of the conductances of the stopcock and the working capillary to be obtained. Such a ratio of the conductances enables us to neglect the possibility of a pressure difference under the working conditions when the bypass stopcock is open. A glass capillary with a length-to-radius ratio of ~ 250 was used, the capillary radius being $(6·46 \pm 0·01) \times 10^{-2} \, \text{cm}$. The capillary was calibrated with mercury; the maximum divergence of the radius from its mean value was less than 0·3 %. Note that the diameter of the compensating tube, which is a necessary element in the relative measurement method, is about 60 mm. The gas temperatures in the 'hot' and 'cold' chambers were 293 °K and 273·2 °K, respectively.
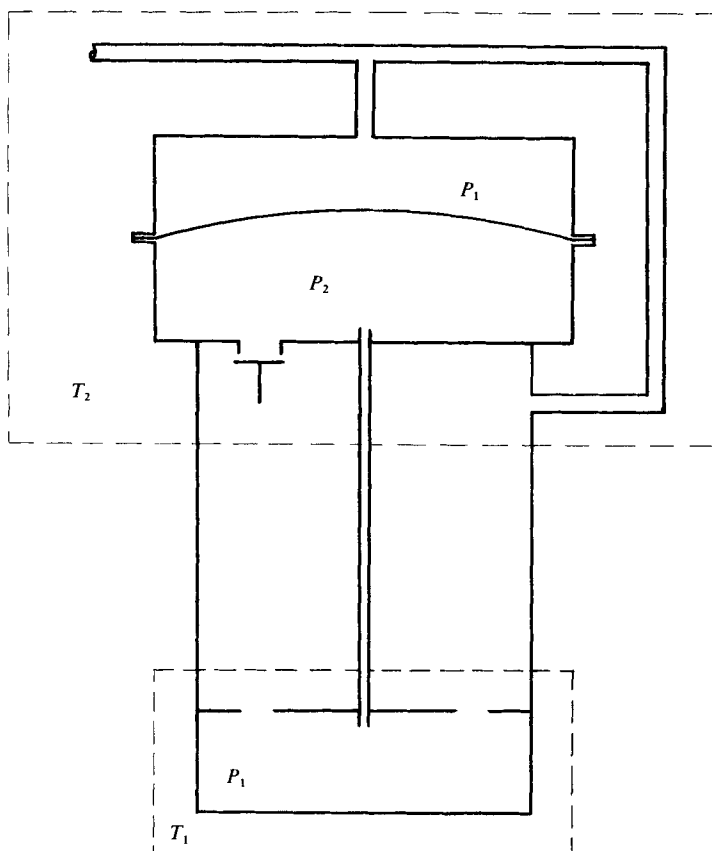
FIGURE 1. Schematic diagram of the experimental set-up.

The quantities measured were the temperature difference ΔT, the temperature T_1 and absolute pressure P_1 of the gas in the 'cold' chamber and the pressure difference $\Delta P = P_2 - P_1$ in the steady-state gas flow in the capillary. The temperature difference was measured by a differential thermocouple with accuracy $\sim 0 \cdot 1\ ^\circ\mathrm{K}$. The pressure P_1 and the pressure difference ΔP were measured by two capacitance micromanometers. The measurements of the t.p.d. effect ΔP were made for $2 \leqslant \delta_P \leqslant 200$ for inert gases (He, Ne, Ar and Xe) and for polyatomic gases (H_2, D_2, N_2, CH_4, CO_2 and SF_6). The purity of the gases was 99·7–99·9 %. The measurement reproducibility was not less than 1 % over the whole range of pressures studied.

4. Measurements and discussion

The measurements of the t.p.d. effect ΔP as a function of the rarefaction parameter δ_P are given in figure 2 for the inert gases and in figure 3 for the polyatomic gases. The parameter δ_P was calculated from the average pressure P_0 and the mean temperature T_0 according to (5c) and (6). As may be seen from figures 2 and 3, the t.p.d. effect attains a maximum in the range $2 \leqslant \delta_P \leqslant 4$. The value attained depends upon the molecular weight of the gas. Note that ΔP decreases when the molecular weight increases.
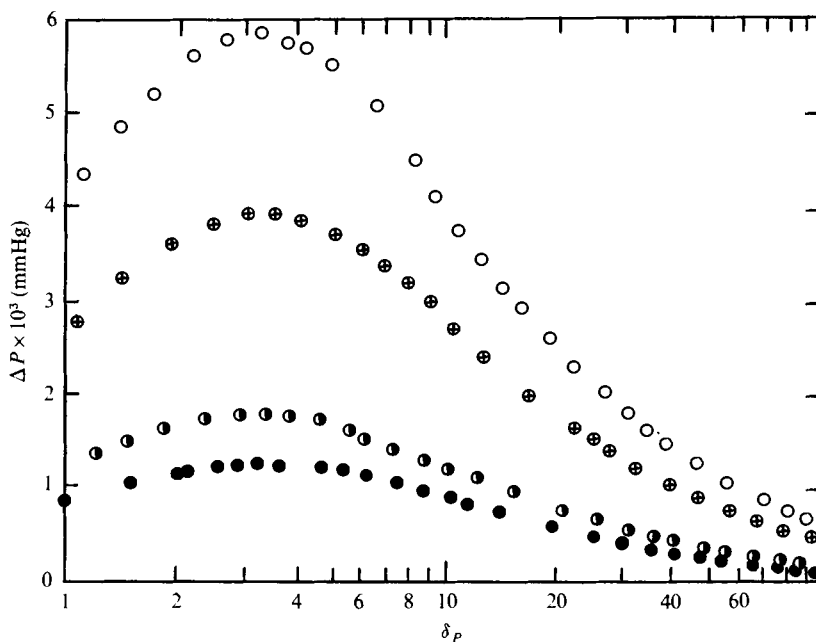
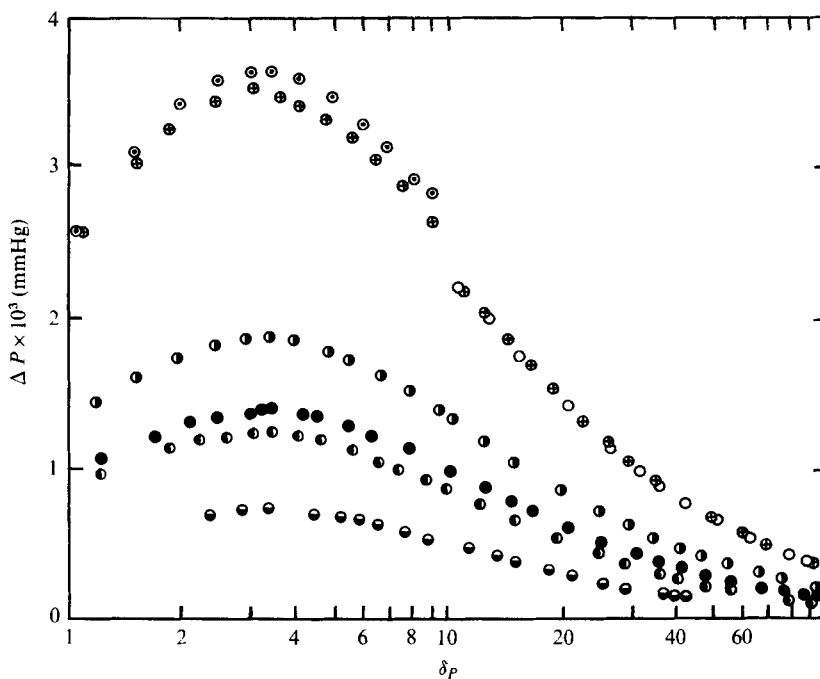FIGURE 2. Dependence of the t.p.d. effect on the rarefaction parameter δ_P for inert gases: $\bigcirc$, He; $\oplus$, Ne; $\mathbb{O}$, Ar; $\bullet$, Xe.

FIGURE 3. Dependence of the t.p.d. effect on the rarefaction parameter δ_P for polyatomic gases: $\bigcirc$, H_2 ($\Delta T \simeq 18.5$ °C); $\odot$, H_2 ($\Delta T \simeq 20.0$ °C); $\oplus$, D_2; $\mathbb{O}$, N_2; $\bullet$, CH_4; $\mathbb{O}$, CO_2; $\ominus$, SF_6.
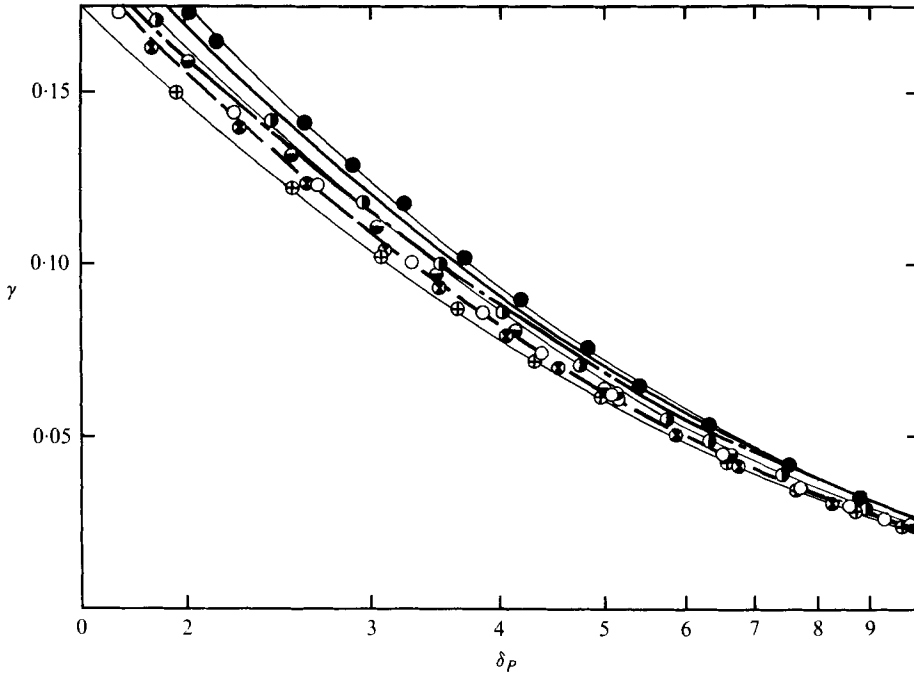
FIGURE 4. Dependence of the index γ on the rarefaction parameter δ_P in the intermediate flow regime. ●, Xe; ◑, Ar; ⊕, Ne; ○, He; ◓, H_2; ◉, CO_2; ——, $\alpha = \epsilon = 1$; – – –, $\alpha = \epsilon = 0.8$; —·—, $\alpha = \epsilon = 0.8$ (Loyalka 1975).

The treatment of the experimental data consisted of the calculation of the index γ of the t.p.d. effect as a function of the rarefaction parameter δ_P from the known pressure difference ΔP, the pressure P_1 in the 'cold' chamber and the temperatures T_1 and T_2 with the help of (4). Note that the results for γ calculated from (4) and (2) differ by less than 0.1 %. This fact justifies the assumptions made in the derivation of (3) and (4). It should be mentioned that the results for γ calculated from (2) and (4) at large temperature differences (77.2 °K and 293 °K; see Borisov *et al.* 1973*a*,*b*) differ significantly. The value of γ in the first case is 13.5–14 % larger than that in the second, this difference being constant over the whole range of pressures investigated. The experimental error in the value of γ is less than 1.5 %.

The experimental values of γ in the intermediate flow regime for some of the gases are shown in figure 4 as a function of the parameter δ_P. The same figure shows the theoretical dependence $\gamma = \gamma(\delta_P)$ obtained from the solution of the integral equations (9*a*, *b*) for the cases $\alpha = \epsilon = 1$ and 0.88. Figure 4 also shows the result of Loyalka (1975) for $\alpha = \epsilon = 0.88$. Note that for $\alpha = \epsilon = 1$ our results coincide with those of Loyalka within 0.01–0.1 %, whereas for $\alpha = \epsilon = 0.88$ they differ by ~ 10 %. In figure 4 the experimental results systematically differ both from each other and from the theory for completely diffuse scattering of molecules at the wall. The reason for this difference is probably the difference in the accommodation coefficients for molecules on the wall. Another possible reason is the energy transfer from the translational to the rotational degrees of freedom resulting from intermolecular collisions, which is characterized by the Eucken factor (Hirschfelder *et al.* 1961, p. 396). However,

Gas	$\dfrac{\gamma_{Xe}}{\gamma}$	$\dfrac{Q_P}{Q_{P_{Xe}}}$	A $\pm \Delta A \times 10^3$	B $\pm \Delta B \times 10^2$	f_{tr} $\pm \Delta f_{tr} \times 10^2$	α $\pm \Delta \alpha \times 10^2$
He	1·13	1·14	1·004 ±2	1·06 ±2	2·49 ±2	0·68 ±3
Ne	1·17	1·17	0·988 ±1	1·00 ±1	2·48 ±1	0·65 ±2
Ar	1·07	1·08	1·030 ±3	1·16 ±3	2·51 ±3	0·74 ±4
Xe	1	1	1·116 ±4	1·42 ±5	2·67 ±5	0·79 ±5
H_2	—	—	0·988 ±2	1·07 ±2	2·40 ±2	0·75 ±3
D_2	—	—	0·963 ±2	0·98 ±2	2·38 ±2	0·69 ±3
N_2	—	—	0·923 ±2	0·92 ±2	2·25 ±3	0·73 ±3
CH_4	—	—	0·958 ±3	1·01 ±4	2·32 ±4	0·75 ±6
CO_2	—	—	0·889 ±4	0·84 ±3	2·19 ±4	0·71 ±5
SF_6	—	—	0·919 ±3	0·76 ±4	2·41 ±4	0·55 ±5

TABLE 3. Parameter values.

this cannot be of prime importance since for inert gases the Eucken factor is close to 2·5.

The best coincidence (within 2 %) of the experimental and theoretical ($\alpha = \epsilon = 1$) results is observed for Xe in the range $\delta_P > 5$. An analogous result was obtained in an experimental investigation of isothermal gas flow in glass capillaries (Porodnov *et al.* 1974). To make a quantitative estimate of the difference in the values of γ for the various gases when $\delta_P \simeq 4$, the ratios γ_{Xe}/γ for all the gases were calculated and are given in table 3. The ratios $Q_P(\epsilon)/Q_{P,Xe}$ of the non-dimensional flow rates of the gases and to that of Xe, taken from Porodnov *et al.* (1974), are also shown in table 3. This table shows that the equation $\gamma_{Xe}/\gamma = Q_P(\epsilon)/Q_{P,Xe}$ is satisfied to within 1 %. This proves that the thermal-creep flow Q_T is independent [see (2)] of the non-isothermal tangential momentum accommodation coefficient α in the range $\delta_P \approx 4$ and confirms the theoretical result obtained in this paper and by Chernjak *et al.* (1973) and Chernjak *et al.* (1975b).

To describe and treat the experimental results in the viscous slip flow regime ($\delta_P > 10$) the formula

$$\gamma = Q_T/Q_P = (A\delta_P^{-1} + B\delta_P^{-2} + C\delta_P^{-3} + D\delta_P^{-4})/Q_P(\epsilon) \tag{12}$$

was used. The Poiseuille flow rate $Q_P(\epsilon)$ in (12) was calculated from the asymptotic formula (10a) and the isothermal tangential momentum accommodation coefficient ϵ was taken from experiments on isothermal flow (Porodnov *et al.* 1974). Figure 5 shows the calculated values of the non-dimensional thermal-creep flow rate Q_T for some of
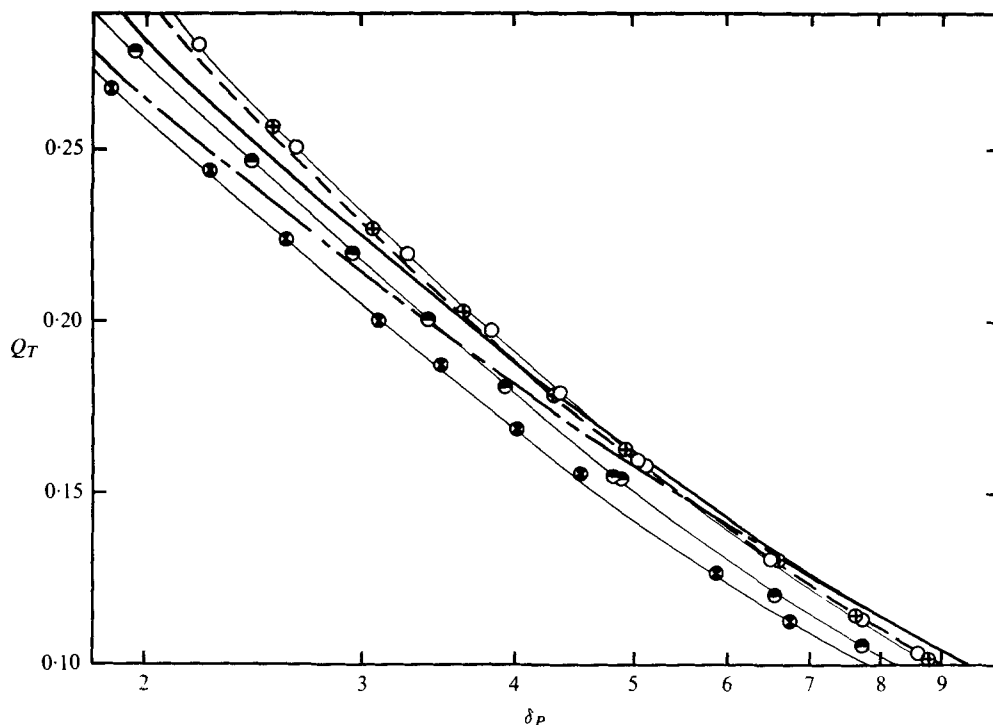
FIGURE 5. Dependence of the non-dimensional thermal-creep flow on the rarefaction parameter δ_P. ◯, He; ⊕, Ne; ◗, N_2; ◑, CO_2; ——, $\alpha = 1$; – – –, $\alpha = 0.8$; –·–, $\alpha = 0.8$ (Loyalka 1975).

the gases as a function of the parameter δ_P, as well as the theoretical values of Q_T for $\alpha = 1$ and $\alpha = 0.8$.

The constants A and B in (12) were determined from the experimental data by the nonlinear least-squares method (on the computer). The constants C and D were calculated from (10a) with $\alpha = 1$ in the first iteration. The calculated values of A and B are given in table 3. Note that the values of A coincide with the results of Annis (1972) within the limits of his experimental errors.

According to (10b), the thermal-creep constants A and B are given by

$$A = \left(\frac{1}{2}+\frac{\alpha}{4}\right)\tfrac{3}{5}f_{\text{tr}}, \quad B = -\frac{2(\pi-1)\,\alpha+(4-\pi)\,\alpha^2}{4\pi^{\frac{1}{2}}}\,(\tfrac{3}{5}f_{\text{tr}})^2. \tag{13}$$

These equations were used to calculate the non-isothermal tangential momentum accommodation coefficient α and the Eucken factor f_{tr} from the known values of the constants A and B. The experimental values of α and f_{tr} are given in table 3. As may be seen, for all the inert gases investigated the Eucken factor is close to 2.5 and does not depend appreciably on the kind of gas. For the polyatomic gases the values of f_{tr} are lower than 2.5 and the accommodation coefficients α are considerably lower than 1.

5. Conclusion

The analysis of the results and their comparison with the experimental and theoretical data show that in the intermediate flow regime ($\delta_P \simeq 4$) the thermal-creep flow depends neither on the non-isothermal tangential momentum accommodation coefficient nor on the proportion of molecules diffusely scattered by the wall. In this flow regime the index γ of the t.p.d. effect depends on the isothermal tangential momentum accommodation coefficient only through the dependence of the Poiseuille flow on this coefficient.

In the viscous slip flow regime ($\delta_P > 10$) the experimental data coincide with the theoretical results for Xe for the case of the complete diffuse scattering of molecules by the wall. It was found, contrary to the results of Itterbeek & Grande (1947), Bennet & Tompkins (1957), McConville *et al.* (1970) and Ganzi & Sandler (1971) that for inert gases the Eucken factor is almost exactly 2·5 (within the limits of the experimental error), as may be expected from the kinetic theory for monoatomic gases, which have no internal degrees of freedom. For polyatomic gases the Eucken factor differs significantly from 2·5, decreasing as the number of internal degrees of freedom of the molecules increases. This result might have been expected from elementary consideration of the intermolecular collision processes.

For all the gases investigated (but particularly the easily condensed gases such as Ar, Xe, CO_2 and SF_6) the non-isothermal tangential momentum accommodation coefficient α essentially differs from unity. Its values differ greatly from those found when isothermal gas flow in capillaries was investigated and are close to the energy accommodation coefficients. This is probably due to the tangential heat transfer in a capillary with thermal creep, when the energy flow accommodation takes place on the wall. The difference between the temperatures of the reflected molecules and the wall can be regarded as a consequence of this. Unfortunately, a number of mathematical difficulties which we are unable to overcome as yet makes the consideration of the influence of this fact on the thermal-creep effect difficult.

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